Out of Equilibrium Transport

across

a Junction of Two Luttinger Liquids

M. A. Cazalilla

The Abdus Salam ICTP

& Brad Marston

Brown University

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OUTLINE

1. MOTIVATION: FUNDAMENTAL AND TECHNOLOGICAL

2. EXPERIMENTS? SOME REAL SYSTEMS

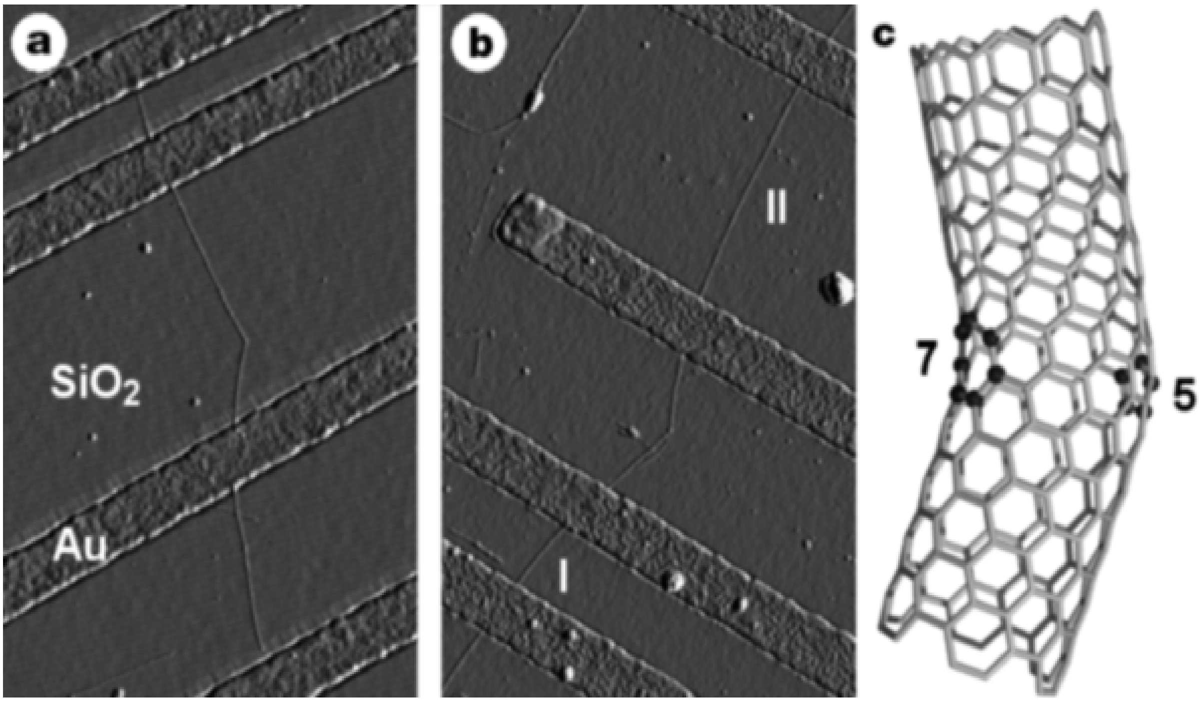
3. TOOL: SOME NUMERICS (DMRG &TdDMRG)

4. TESTING TdDMRG

5. RESULTS

6. OUTLOOK

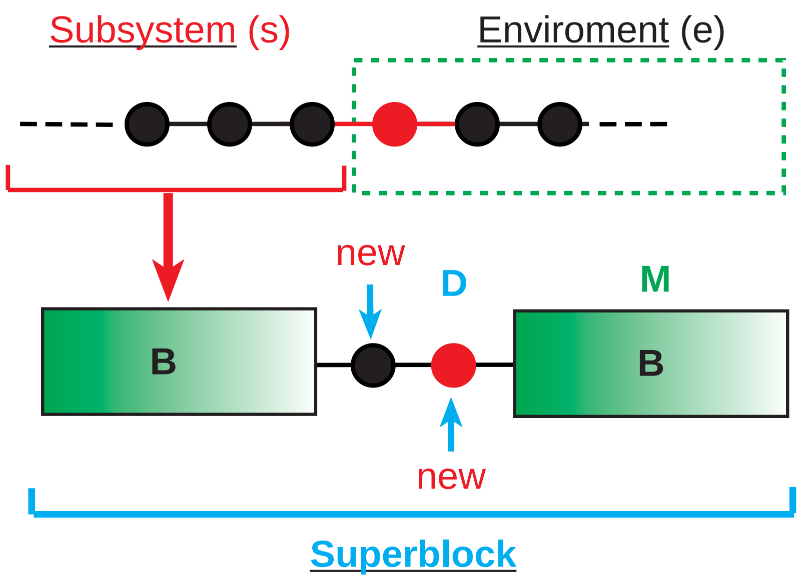
SOME REAL SYSTEMS: JUNCTION OF TWO SWCNT



See Zhen Yao et al.Nature 402, 273 (1999)

THE TOOL: WHAT IS DMRG?

Problem: $\dim(\mathcal{H}) \sim D^L$



1. Diagonalize H for $L = 4$

$$H \Psi_{se} = E_o \ \Psi_{se}.$$

2. Form & Diagonalize reduced density matrices:

$$\rho_{ss'} = \sum_e \ \Psi^*_{se} \Psi_{s'e}$$

3. If dim(Block)$> M \Rightarrow$ TRUNCATE! Project $H \Rightarrow H_{trunc}$.
4. Add two new sites: $L \to L + 2$. Iterate 1,2,3, & 4 up to $L_{\max}$.
5. Increase M for systematic improvement.

See S.R. White, PRB 48, 10345 (1993)

Time-dependent DMRG

- Run DMRG & obtain $\Psi_0^{\rm trunc}$, E_0, and $H_{\rm trunc}$.
- Integrate time-dependent Schrödinger eq. forward in time

$$i\hbar\frac{d\Psi(t)}{dt} = \left[H_{trunc} + H'(t) - E_0\right]\Psi(t); \quad \Psi(0) = \Psi_0^{\rm trunc}.$$

- Compute observables. E.g. the current across the central link:

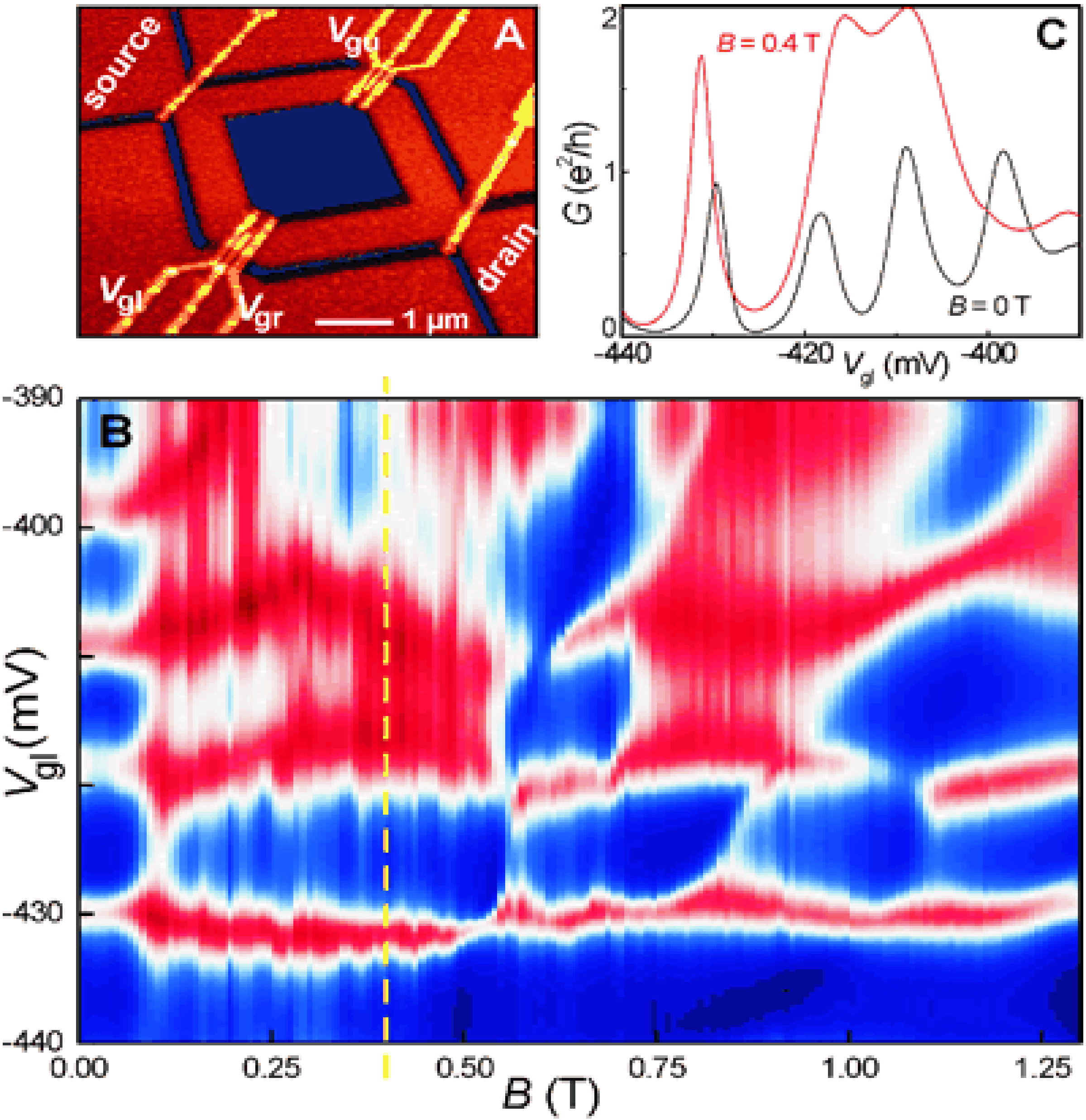
$$J(t) = -\frac{2e}{\hbar}{\rm Re}\{i\ t_q\ \langle\Psi(t)|c_q^\dagger c_{q-1}|\Psi(t)\rangle\}\ .$$

- For illustrative purposes , we shall use a pulsed bias:

$$H'(t) = \delta\mu_0\,\theta(t - t_0)\,[N_{\rm L} - N_{\rm R}]\ .$$

See M. A. Cazalilla & J. B. Marston, cond-mat/0109158

QUANTUM DOT



See W. G. var der Wiel et al. Science 289, 2105 (2000)

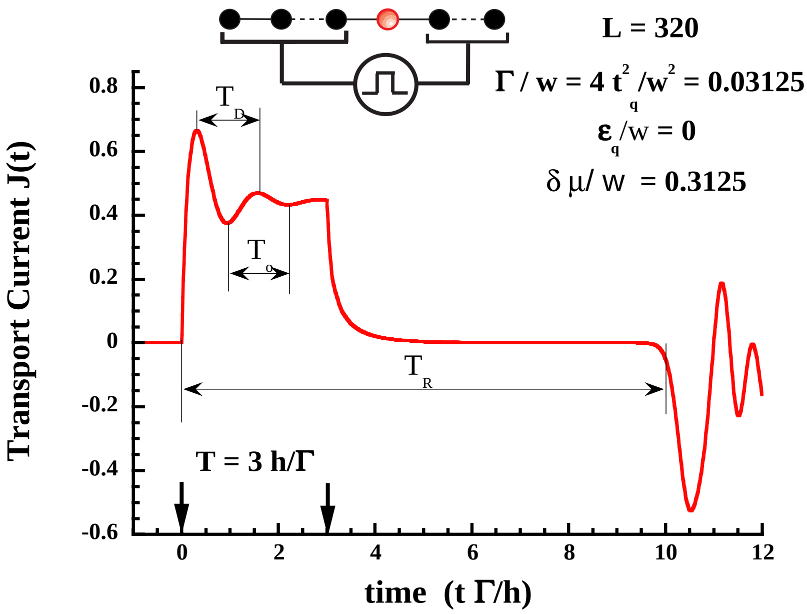
THE TEST: TIME SCALES AT FINITE SIZE

Tunneling of spinless electrons through a quantum dot:

$$(j = 1 \ldots L, \;\; q = \frac{L}{2} + 1)$$

$$H \;=\; -\frac{w}{2} \sum_{j \neq q-1,q} \left[c_{j+1}^{\dagger} c_j + \text{H.c.}\right] + \epsilon_q n_q$$
$$-\;\; t_q \left[c^{\dagger}(q) c_{\sigma}(q-1) + c^{\dagger}(q+1) c_{\sigma}(q) + \text{H.c.}\right].$$

Response to a pulsed bias: Exact Solution at finite size



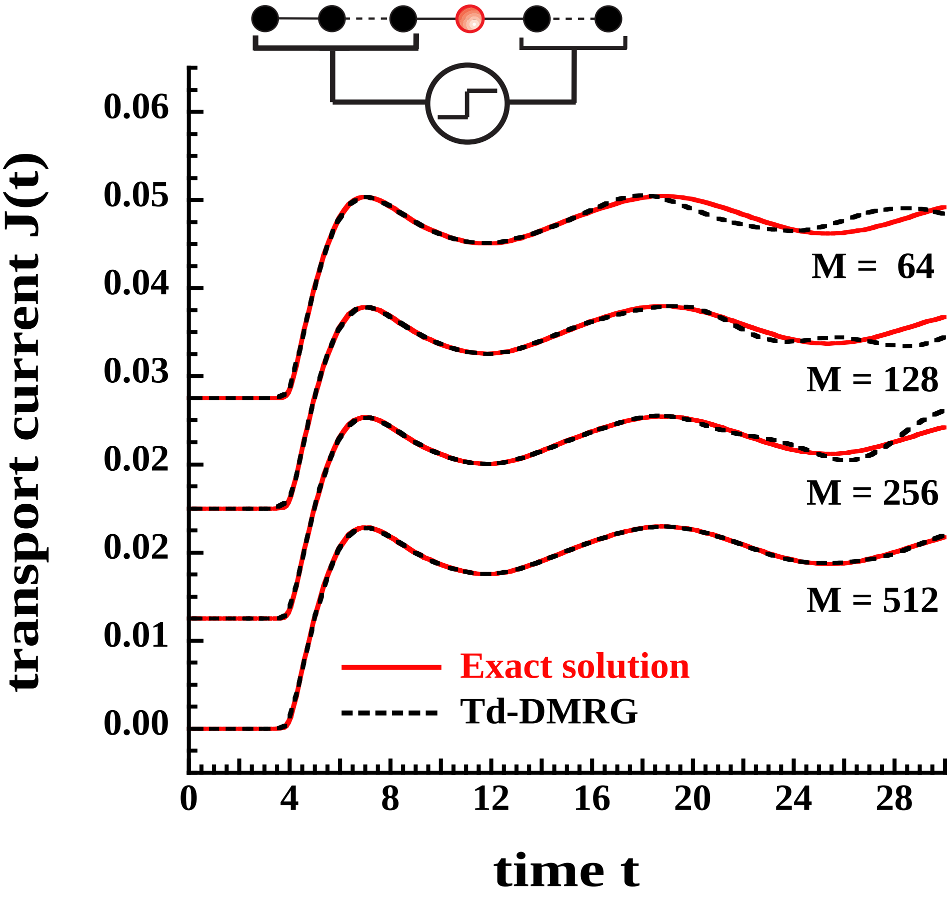
Relevant time scales:

- Ringing $T_o = 4\pi\hbar/|2\delta\mu_o - (\epsilon_q - \delta\mu_o)| \approx 1.26\hbar/\Gamma$, Decay $T_d \sim \hbar/\Gamma$
- Reflection Time: $T_r \approx L/v_F = 10\hbar/\Gamma$

See N. S. Wingreen *et al.*, PRB 48, 10345 (1993).

THE TEST: TdDMRG vs. EXACT

- Chain Length $L = 64$
- Quantum dot parameters $t_q/w = 0.15$ and $\epsilon_q/w = -0.25$
- Pulse parameters $\delta\mu_o/w = -0.25$ and $t_o = 4\tau_w$
- Time in units of $\tau_w = \hbar/w$. Current in units of e/τ_w

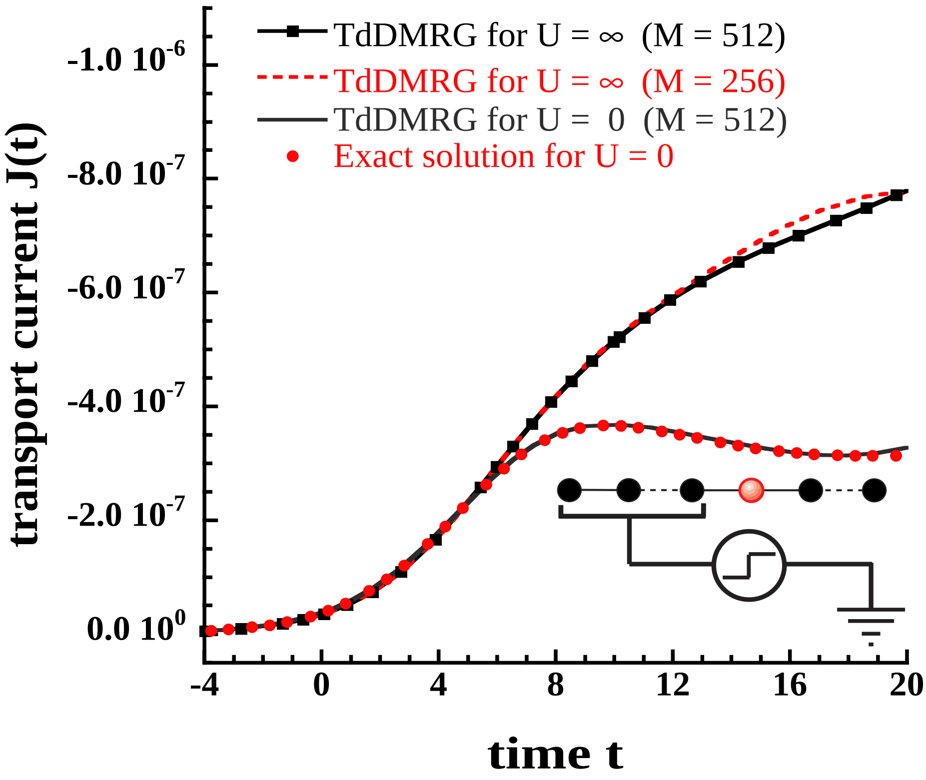


Relevant time scales:

- Ringing $T_o = 4\pi\hbar/|2\delta\mu_o - (\epsilon_q - \delta\mu_o)| = 4\pi\tau_w$
- Reflection Time: $T_R \approx L/v_F = 32\tau_w$

ANDERSON MODEL: "REAL" Q. DOTS

$$H_{\rm qdot} = -\frac{w}{2} \sum_{\sigma, j\neq q-1,\, q} \left[c^{\dagger\sigma}_{j+1} c_{\sigma j} + \text{H.c.}\right] + \varepsilon_q\, n_q + U\, n_{\uparrow q} n_{\downarrow q}$$
$$- \; t_q \sum_{\sigma} \left[c^{\dagger\sigma}_{q} c_{\sigma q-1} + c^{\dagger\sigma}_{q+1} c_{\sigma q} + \text{H.c.}\right],$$



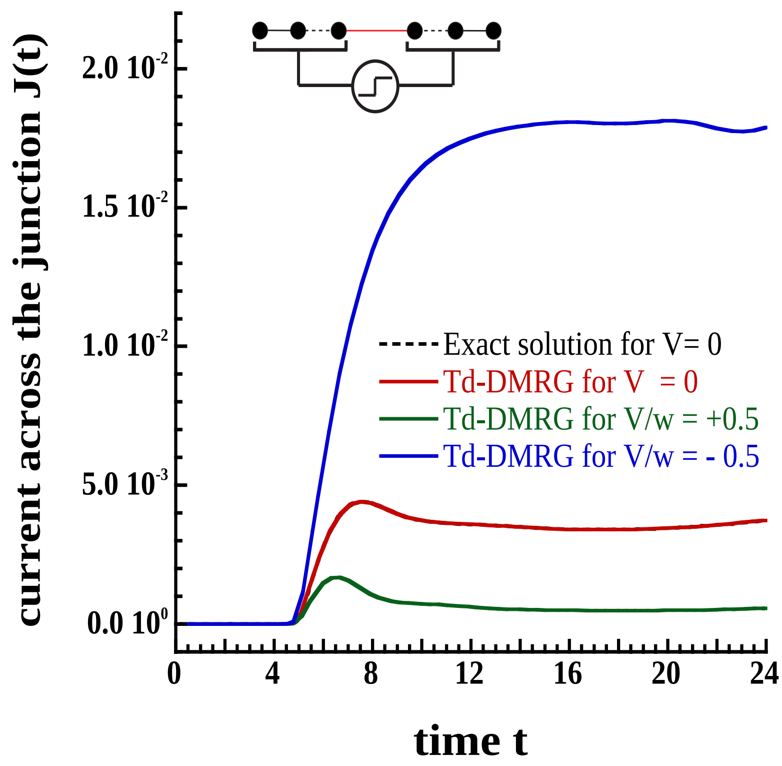
(Bias is applied to the left lead only)

RESULTS: TUNNELING BETWEEN LL's

Junction Hamiltonian ($|V| < w \Rightarrow$ the leads are LL's):

$$H = -\frac{w}{2}\sum_{j\neq q}\left[c^\dagger_{j+1}c_j + \text{H.c.}\right] + V\sum_{j\neq q}\left[n_j - \frac{1}{2}\right]\left[n_{j+1} - \frac{1}{2}\right]$$
$$- \quad t_q\left[c^\dagger_q c_{q-1} + \text{H.c.}\right]$$

- Chain Length $L = 64$
- Junction hopping $t_q/w = 0.125$
- Pulse parameters $\delta\mu_0/w = 0.0625$ and $t_0 = 5\tau_w$



C.L. Kane & M.P.A. Fisher, PRB 46, 15233 (1992).

RESULTS: TUNNELING BETWEEN CDW INSULATORS

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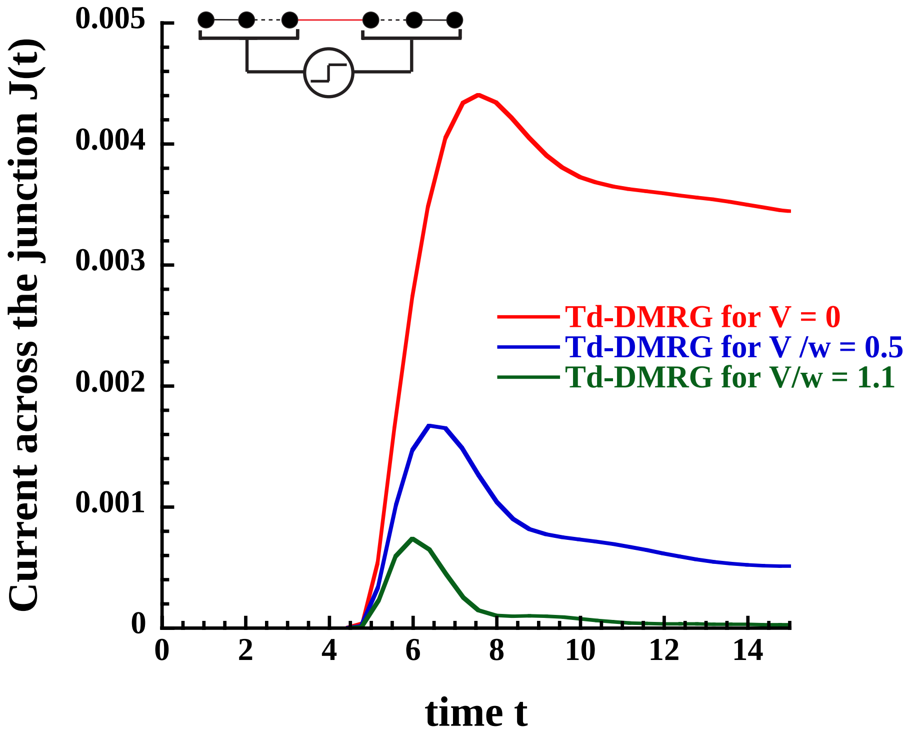

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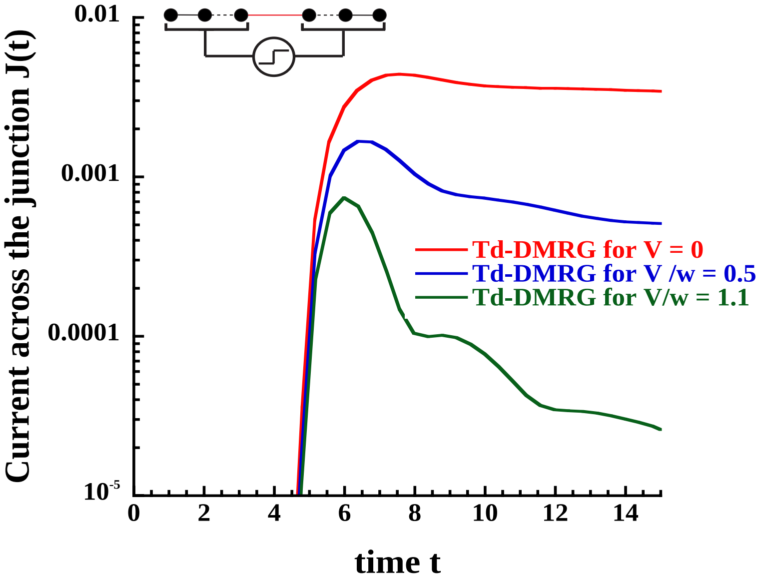

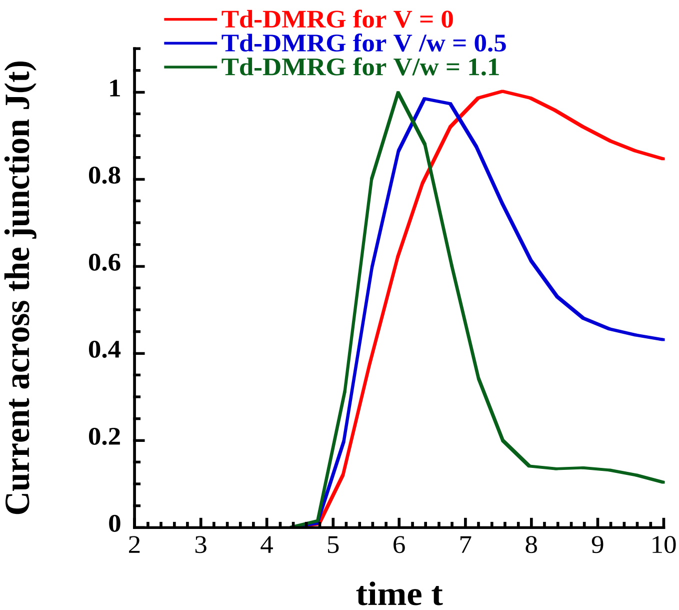

CONCLUSIONS & OUTLOOK

- TdDMRG: Systematic & approximation independent.
- It can help improving existing models/approximations.
- Qualitative agreement: Theory needs to be worked out.
- “Weak link” limit: Bosonization?
- Comparison to Experiments?